Z_k-Magic Labeling of Snake Related Graphs

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ABSTRACT

For any non-trivial abelian group A under addition a graph G is said to be A-magic if there exists a labeling f: E(G)→A − {0} such that, the vertex labeling f^+ defined as $f^+(v) = \sum f(uv)$ taken over all edges uv incident at vis a constant. An A-magic graph G is said to be Z_k-magic graph if the groupA is Z_k the group of integers modulo k. These Z_k-magic graphs are referred to as k-magic graphs. In this paper we prove that the graphs such as triangular snake, double triangular snake, $T^2(T_n)$ and quadrilateral snakeare Z_k-magic graphs.

Keywords: *A-magic labeling, Z_k-magic labeling, Z_k-magic graph.*

AMS Subject Classification (2010): 05C78

1. Introduction

Graph labeling is currently an emerging area in the research of graph theory. A graph labeling is an assignment of integers to vertices or edges or both subject to certain conditions. A detailed survey was done by Gallian in [8]. If the labels of edges are distinct positive integers and for each vertex v the sum of the labels of all edges incident with v is the same for every vertex v in the given graph then the labeling is called a magic labeling. Sedlacek [11] introduced the concept of A-magic graphs. A graph with real-valued edge labeling such that distinct edges have distinct non-negative labels and the sum of the labels of the edges incident to a particular vertex is same for all vertices. Low and Lee [10] examined the A-magic property of the resulting graph obtained from the product of two A-magic graphs. Shiu, Lam and Sun [12] proved that the product and composition of A-magic graphs were also A-magic. For any non-trivial abelian group A under addition a graph G is said to be A-magic if there exists a labeling f: E(G)→A−{0} such that, the vertex labeling f^+ defined as $f^+(v) = \sum f(uv)$ taken over all edges uv incident at v is a constant. An A-magic graph G is said to be Z_k-magic graph if the group A is Z_k the group of integers modulo k. These Z_k-magic graphs are referred to as k-magic graphs. Shiu and Low [13] determined all positive integers k for which fans and wheels have a Z_k-magic labeling with a magic constant 0. Kavitha and Thirusangu [9] obtained a Z_k-magic labeling of two cycles with a common vertex. Motivated by the concept of A-magic graph in [11] and the results in [10, 12, 13] Jeyanthi and Jeya Daisy [1-7] proved that some standard graphs admit Z_k-magic labeling. We use the following definitions in the subsequent section.

Definition 1.1 The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3.

Definition 1.2 A double triangular snake DT_n consists of two triangular snakes that have a common path.

Definition 1.3 A quadrilateral snake Q_n is obtained from a path P_n by replacing each edge of P_n by a cycle C_4.

Definition 1.4 Let G = (V,E) be a simple graph and $G' = (V', E')$ be another copy of graph G. Join each vertex v of G to the corresponding vertex v' of G' by an edge. The new graph thus obtained is the 2- tuple graph of G. 2- tuple graph of G is denoted by T^2 (G). Further if G = (p, q) then $V\{T^2(G)\} = 2p$ and $E\{T^2(G)\} = 2q+p$.

2 Z_k-Magic Labeling of Snake Related Graphs

In this section we prove that the graphs such as triangular snake, double triangular snake, $T^2(T_n)$ and quadrilateral snake are Z_k-magic graphs.

Theorem 2.1

The triangular snake T_n is Z_k magic for all $n \geq 2$.

Proof:

Let the vertex set of T_n be $V(T_n) = \{v_i / 1 \leq i \leq n\} \cup \{u_i / 1 \leq i \leq n-1\}$

and the edge set be $E(T_n) = \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_i u_i / 1 \leq i \leq n-1\}$

We consider the following two cases.

Case (i): n is odd.

For any integer 'a' such that k > a.

Define the edge labeling $f : E(T_n) \to Z_k - \{0\}$ as follows:

$$f(v_i v_{i+1}) = \begin{cases} a & \text{for } i \text{ is odd} \\ k-a & \text{for } i \text{ is even} \end{cases}$$

$$f(v_i u_i) = k-a \quad \text{for } 1 \leq i \leq n-1$$

$$f(u_i v_{i+1}) = a \quad \text{for } 1 \leq i \leq n-1.$$

Then the induced vertex labeling $f^+ : V(T_n) \to Z_k$ is $f^+(v) \equiv 0 (\text{mod } k)$ for all $v \in V(T_n)$.

Case (ii): n is even.

For any integer 'a' such that k > 2a.

Define the edge labeling $f : E(T_n) \to Z_k - \{0\}$ as follows:

$$f(v_i v_{i+1}) = \begin{cases} a & \text{for } i \text{ is odd} \\ k-a & \text{for } i \text{ is even} \end{cases}$$

$$f(v_i u_i) = a \quad \text{for } 1 \leq i \leq n-1$$

$f(u_i v_{i+1}) = a \;\; for \; 1 \le i \le n-1.$

Then the induced vertex labeling $f^+ : V(T_n) \to Z_k$ is $f^+(v) \equiv 2a (\text{mod } k)$ for all $v \in V(T_n)$.

Thus triangular snake T_n is Z_k magic for all $n \ge 2$.

Examples of Z_7-magic labeling of T_5 and Z_4-magic labeling of T_6 are shown in Figure 1.

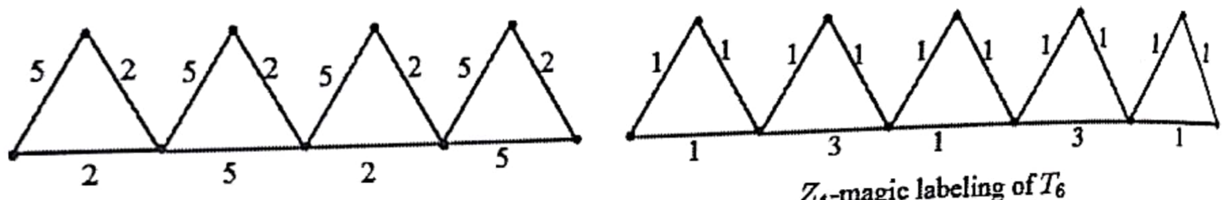

Figure 1

Theorem 2

The Double triangular snake DT_n is Z_k magic when n is odd.

Proof:

Let the vertex set of DT_n be $V(DT_n) = \{v_i / 1 \le i \le n\} \cup \{u_i, w_i / 1 \le i \le n-1\}$ and the edge set be $E(DT_n) = \{v_i v_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_{i+1}, w_i v_{i+1} / 1 \le i \le n-1\} \cup \{v_i u_i, v_i w_i / 1 \le i \le n-1\}$

For any integer 'a' such that $k > 2a$.

Define the edge labeling $f : E(DT_n) \to Z_k - \{0\}$ as follows:

$$f(v_i v_{i+1}) = \begin{cases} 2a & for \; i \; is \; odd \\ k-2a & for \; i \; is \; even \end{cases}$$

$f(v_i u_i) = k - a \;\; for \; 1 \le i \le n-1$

$f(v_i w_i) = k - a \;\; for \; 1 \le i \le n-1$

$f(w_i v_{i+1}) = a \;\; for \; 1 \le i \le n-1$

$f(u_i v_{i+1}) = a \;\; for \; 1 \le i \le n-1.$

Then the induced vertex labeling $f^+ : V(DT_n) \to Z_k$ is $f^+(v) \equiv 0 (\text{mod } k)$ for all $v \in V(DT_n)$.

Thus the double triangular snake DT_n is Z_k magic when n is odd.

An example of Z_7-magic labeling of DT_5 is shown in Figure 2.

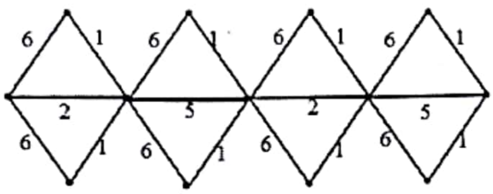

Figure 2 Z_7-magic labeling of DT_5

Theorem 3

The 2-tuple graph of triangular snake $T^2(T_n)$ is Z_k magic for all $n \geq 2$.

Proof:

Let the vertex set of $T^2(T_n)$ be $V(T^2(T_n)) = \{v_i, v_i' / 1 \leq i \leq n\} \cup \{u_i, u_i' / 1 \leq i \leq n-1\}$

and the edge set be

$E(T^2(T_n)) = \{v_i v_{i+1}, v_i' v_{i+1}', v_i u_i, v_i' u_i', u_i u_{n-i}' / 1 \leq i \leq n-1\} \cup \{u_i v_{i+1}, u_i' v_{i+1}' / 1 \leq i \leq n-1\} \cup \{v_i v_{n-i+1}' / 1 \leq i \leq n\}$.

For any integer a such that k > 4a.

Define the edge labeling $f : E(T^2(T_n)) \to Z_k - \{0\}$ as follows:

$f(v_i v_{i+1}) = f(v_i' v_{i+1}') = k - a \;\; for \; 1 \leq i \leq n-1$

$f(v_i u_i) = f(v_i' u_i') = k - a \;\; for \; 1 \leq i \leq n-1$

$f(u_i v_{i+1}) = f(u_i' v_{i+1}') = k - a \;\; for \; 1 \leq i \leq n-1$

$f(v_1 v_n') = f(v_n v_1') = 2a$

$f(v_i v_{n-i+1}') = 4a \;\; for \; 2 \leq i \leq n-1$

$f(u_i u_{n-i}') = 2a \;\; for \; 1 \leq i \leq n-1.$

Then the induced vertex labeling $f^+ : V(T^2(T_n)) \to Z_k$ is $f^+(v) \equiv 0 (\mathrm{mod}\; k)$ for all $v \in V(T^2(T_n))$. Thus the 2-tuple graph of triangular snake $T^2(T_n)$ is Z_k magic for all $n \geq 2$.

An example of Z_8-magic labeling of $T^2(T_3)$ is shown in Figure 3.

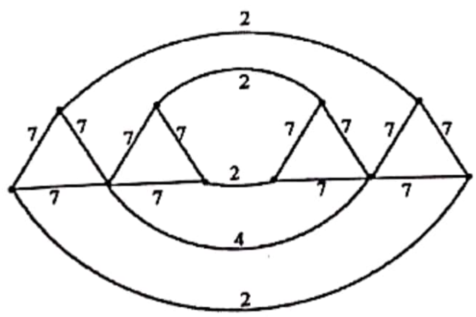

Figure 3 Z_8-magic labeling of $T^2(T_3)$

Theorem 2.4

The quadrilateral snake Q_n is Z_k magic for all $n \geq 2$.

Proof:

Let the vertex set of Q_n be $V(Q_n) = \{v_i / 1 \leq i \leq n\} \cup \{u_i / 1 \leq i \leq n-1\} \cup \{w_i / 1 \leq i \leq n-1\}$

and the edge set be

$E(Q_n) = \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{w_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_i u_i,\ u_i w_i / 1 \leq i \leq n-1\}$

For any integer a such that $k > a$.

Define the edge labeling $f : E(Q_n) \to Z_k - \{0\}$ as follows:

$$f(v_i v_{i+1}) = \begin{cases} k-a & \text{for } i \text{ is odd} \\ a & \text{for } i \text{ is even} \end{cases}$$

$$f(u_i w_i) = \begin{cases} k-a & \text{for } i \text{ is odd} \\ a & \text{for } i \text{ is even} \end{cases}$$

$$f(u_i v_i) = \begin{cases} a & \text{for } i \text{ is odd} \\ k-a & \text{for } i \text{ is even} \end{cases}$$

$$f(w_i v_{i+1}) = \begin{cases} a & \text{for } i \text{ is odd} \\ k-a & \text{for } i \text{ is even.} \end{cases}$$

Then the induced vertex labeling $f^+ : V(Q_n) \to Z_k$ is $f^+(v) \equiv 0 \pmod{k}$ for all $v \in V(Q_n)$.

Thusthe quadrilateral snake Q_n is Z_k magic for all $n \geq 2$.

An example of a Z_5-magic labeling of Q_4 is shown in Figure 4.

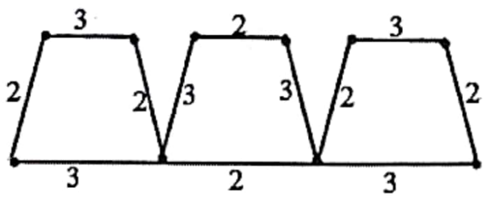

Figure 4 Z_5**-magic labeling of** Q_4

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